



Optimal allocation and sizing of capacitors to minimize the transmission line loss and to improve the voltage profile

I. Ziari^{a,*}, G. Ledwich^a, A. Ghosh^a, D. Cornforth^b, M. Wishart^a

^a Queensland University of Technology, Brisbane, Australia

^b CSIRO Energy Technology, Newcastle NSW, Australia

ARTICLE INFO

Keywords:

Allocation and sizing
Capacitor
Distribution system
DPSO

ABSTRACT

In this paper, a modified discrete particle swarm optimization is presented to find the optimal placement and size of capacitors in a distribution system. The objective function is composed of the line loss and the capacitors investment cost. The bus voltage and the feeder current as constraints are included in the objective function by a constraint penalty factor.

To validate the proposed method, the 18-bus IEEE distribution system and the semi-urban distribution system which is connected to bus 2 of the Roy Billinton test system are used. The proposed method is applied to the problem and its robustness and accuracy are studied. The results are compared with pure DPSO, genetic algorithm, and nonlinear programming. It is illustrated in two examples that the proposed optimization method is more accurate and particularly more robust than others for the planning of capacitors.

© 2010 Elsevier Ltd. All rights reserved.

1. Introduction

As the electrical loads are growing, the distribution system is developing to supply the customers with higher levels of demand. Growing the loads increases the lines current leading to the increase of loss. This also decreases the voltage level in the distribution network. Capacitors are used commonly to solve these difficulties. However, the investment cost is an issue which prevents their wide use. This highlights the importance of optimal allocation and sizing of the capacitors (OASC).

Given the discrete nature of the allocation and sizing problem, local minima are the main difficulty in the OASC problem. There are several publications on solving the OASC problem. These methods are categorized into two main groups: analytical-based methods [1–5] and heuristic-based methods [6–12]. The analytical methods have low computation time, but they do not deal appropriately with the local minima. For solving the local minima issue, the heuristic methods are extensively applied in the literatures [6–12].

A nonlinear programming package is employed in [1] for finding the location and size along with scheduling of capacitors to minimize the loss and to improve the voltage profile. Wu et al. in [2] employ the maximum sensitivities selection method for allocation of fixed and switched capacitors in a distorted substation voltage. Similarly in 1997, Chao et al. in [3] employ the sensitivity analysis for planning of the fixed and switched capacitors to minimize the loss and to improve the voltage profile. In [4], a two-phase approach is presented for solving the OASC problem. The capacitors size is assumed to be a continuous variable in the first phase and the optimization is performed using a conic optimization method, and then, the mixed integer programming is used to make the problem more practical by considering the discrete capacitor size. The optimization problem is formulated as a mixed-integer linear problem after a linearization in [5], and then, the mixed-integer linear programming is used for solving the problem.

The Genetic Algorithm (GA) as a heuristic-based method is studied and used in [6] for finding the optimal placement of capacitors. This optimization method along with a fast algorithm for computing the energy loss is presented in [7] for

* Corresponding author.

E-mail address: i.ziari@qut.edu.au (I. Ziari).

planning the capacitors. Masoum et al. in [8] use GA for optimal placement, replacement and sizing of the capacitors with consideration of nonlinear loads. These authors use the combination of GA and Fuzzy Logic in [9] and improve on the results obtained by GA. A hybrid method, composed of a modified differential evolution and integer programming, is proposed for solving the OASC problem in [10]. The results of this approach are compared with the hybrid differential evolution, simulated annealing, and ant system. Another heuristic method, Ant Colony Search algorithm, is used in [11] for reconfiguration and finding the placement of capacitors to reduce the line loss. Simulated Annealing (SA) as a heuristic method is employed in [12] for scheduling of the main transformer under load tap changers and the shunt capacitors in distribution systems.

In this paper, a heuristic method called the Discrete Particle Swarm Optimization (DPSO) algorithm is employed to find the optimal placement and size of capacitors to minimize the line loss and to improve the voltage profile. To deal appropriately with the local minima problem as the main issue in the OASC problem, the DPSO is modified by increasing the diversity of the optimizing variables. For this purpose, the mutation and crossover operators are applied to the optimizing variables. The results illustrate that the Modified DPSO (MDPSO) is more robust and more accurate compared with other methods for capacitor planning. The objective function is composed of the line loss and the capacitors investment cost. The bus voltage and the feeder current are constraints which should be maintained within standard levels.

In Section 2, the allocation and sizing of capacitors are formulated. The optimization algorithm and its implementation are explained in Section 3. The results and conclusions are given in Sections 4 and 5.

2. Problem formulation

The loads and capacitors are modelled as impedance, a series RL for loads and a capacitive reactance for capacitors. The objective function and the constraints are also expressed in this section.

2.1. Load and capacitor model

As mentioned before, the loads and capacitors are modelled as impedance. The impedance models used in this paper for loads and capacitors are given in (1) and (2):

$$Z_{Load_i} = R_{Load_i} + jX_{Load_i} \quad i = 1, 2, 3, \dots, NL \quad (1)$$

where NL is the number of Loads, Z_{Load_i} is the load impedance in load i , R_{Load_i} is the load resistance in load i , and X_{Load_i} is the load reactance in load i .

$$Z_{C_k} = -jX_{C_k} \quad k = 1, 2, 3, \dots, NC \quad (2)$$

where NC is the number of capacitors, Z_{C_k} is the capacitor impedance in capacitor k , and X_{C_k} is the capacitor reactance in capacitor k .

2.2. Objective function and constraints

Minimizing the total cost of capacitors as well as the distribution line loss is the main objective of the OASC problem. The bus voltage and the feeder current as constraints are included in the objective function with a penalty factor. As all of the objective function elements are simply converted into the composite equivalent cost, this problem is solved using a single-objective optimization method. The objective function is defined as follows:

$$OF = C_{CAPITAL} + \sum_{t=1}^T \frac{C_{O\&M} + C_{LOSS}}{(1+r)^t} + \lambda \quad (3)$$

where $C_{CAPITAL}$ and $C_{O\&M}$ are the capital cost and the operation and maintenance cost of capacitors, C_{LOSS} is the line loss cost, r is the discount rate, T is the number of years in the study timeframe, and λ is the constraint penalty factor.

The line loss can be converted into an equivalent cost as

$$C_{LOSS} = k_L \cdot P_{LOSS} \quad (4)$$

where k_L is the cost per MWh and P_{LOSS} is the line loss.

The bus voltage and the feeder current should be maintained within standard levels as given in (5) and (6):

$$0.95pu \leq V_{bus} \leq 1.05 \quad (5)$$

$$I_f \leq I_f^{rated} \quad (6)$$

where V_{bus} is the actual bus voltage, and I_f and I_f^{rated} are the actual and rated feeder current, respectively.

3. Methodology

3.1. Overview of PSO

PSO is a population-based and self-adaptive technique introduced originally by Kennedy and Eberhart in 1995 [13]. This algorithm handles a population of individuals in parallel to probe search areas of a multi-dimensional space where the

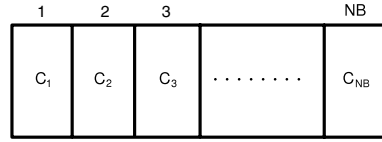


Fig. 1. Structure of a particle.

optimal solution is searched. The individuals are called *particles* and the population is called a *swarm*. Each particle in the swarm moves towards the optimal point with an adaptive velocity [14].

Mathematically, particle i in an n -dimensional vector is represented as $X_i = (x_{i,1}, x_{i,2}, \dots, x_{i,n})$. The velocity of this particle is also an n -dimensional vector as $V_i = (v_{i,1}, v_{i,2}, \dots, v_{i,n})$. Alternatively, the best position related to the lowest value of the objective function for each particle is represented as $Pbest_i = (pbest_{i,1}, pbest_{i,2}, \dots, pbest_{i,n})$ and the global best position among all particles or best $pbest$ is denoted as $Gbest_i = (gbest_{i,1}, gbest_{i,2}, \dots, gbest_{i,n})$. During the optimization procedure, the velocity and position of particles are updated iteratively [14].

The DPSO, the discrete version of PSO, is an optimization method which is applied to the discrete problems like OASC where the particles are optimizing variables such as the capacitor size which are analysed as integer values. In this situation, the optimal solution can be achieved by rounding off the actual particle value to the nearest integer value during the iterations. In [14], it is mentioned that the performance of the DPSO is not influenced by this rounding off process. Note that the continuous methods perform the rounding off after the convergence of the algorithm, while in DPSO, it is applied to all particles during each iteration of the optimizing procedure.

3.2. Applying hybrid DPSO to problem

The first step in an optimization procedure is identifying the optimizing variables. The optimizing variables, particles, in the OASC problem are the size and the placement of capacitors. Fig. 1 shows the structure of particles in the employed DPSO.

As observed, the particle is composed of NB cells with the value of C_i . Each candidate bus for installing a capacitor is assigned by a cell and the rating of the capacitor at the relative candidate bus is the value of the corresponding cell. For example, C_i is the rating of the capacitor installed at bus i . If all buses are candidates for installing capacitors, NB will be the number of buses. Therefore, the number of optimizing variables is at most equal to the number of buses. If the value of a cell, the capacitor size, is more than a specific threshold, it indicates that a capacitor is installed at that bus. Otherwise, no capacitor is placed at the relative bus. This specific threshold is the minimum size of the available set of capacitors.

It will be observed that the results obtained by pure DPSO are improved when the crossover and mutation operators are included in the DPSO procedure. Furthermore, the robustness of the optimization method is improved by this modification. This is mainly because these operators increase the diversity of optimizing variables. Fig. 2 shows the flowchart of the proposed method. The description and comments of the steps are presented as follows.

Step 1. (Input system data and initialization)

In this step, the distribution network configuration and data and the available capacitors are input. The maximum allowed voltage drop and the characteristics of feeders, impedance and rated current, are also specified. The DPSO parameters, number of population and iterations, as well as the PSO weight factors are also identified. The random-based initial population of particles X_j (size of capacitors) and the particles velocity V_j in the search space are also initialized.

Step 2. (Calculate the objective function)

Given the capacitors size determined in the previous step, the admittance matrix is reconstructed. Using the new admittance matrix, a load flow is run and the buses voltage and the feeders current are calculated. These are used to calculate the distribution line loss.

The objective function is now constituted by (1). The constraints are also computed using (3) and (4) in this step and included in the objective function with a penalty factor. It means that if a constraint is not satisfied, a large number as a penalty factor is added to the objective function to exclude the relevant solution from the search space.

Step 3. (Calculate $pbest$)

The component of the objective function value associated with the position of each of the particles is compared with the corresponding value in previous iteration and the position with the lower objective function is recorded as $pbest$ for the current iteration:

$$pbest_j^{k+1} = \begin{cases} pbest_j^k & \text{if } OF_j^{k+1} \geq OF_j^k \\ x_j^{k+1} & \text{if } OF_j^{k+1} < OF_j^k \end{cases} \quad (7)$$

where, k is the number of iterations, and OF_j is the objective function component evaluated for particle j .

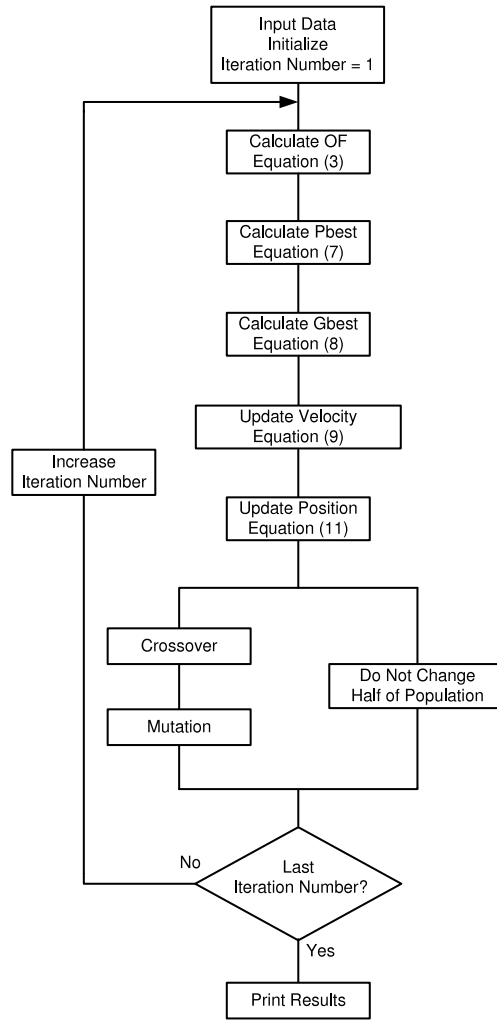


Fig. 2. Algorithm of the proposed PSO-based approach.

Step 4. (Calculate gbest)

In this step, the lowest objective function among the *pbests* associated with all particles in the current iteration is compared with it in the previous iteration and the lower one is labelled as *gbest*:

$$gbest^{k+1} = \begin{cases} gbest^k & \text{if } OF^{k+1} \geq OF^k \\ pbest_j^{k+1} & \text{if } OF^{k+1} < OF^k. \end{cases} \quad (8)$$

Step 5. (Update position)

The position of particles for the next iteration can be calculated using the current *pbest* and *gbest* as follows:

$$V_j^{k+1} = \omega V_j^k + c_1 \text{rand}(pbest_j^k - X_j^k) + c_2 \text{rand}(gbest^k - X_j^k) \quad (9)$$

where V_j^k is the velocity of particle j at iteration k , ω is the inertia weight factor, c_i is the acceleration coefficients, X_j^k is the position of particle j at iteration k , $pbest_j^k$ is the best position of particle j at iteration k , and $gbest^k$ is the best position among all particles at iteration k .

As mentioned before, using the available data, ω as inertia weight factor, and c_1 and c_2 as acceleration coefficients, the velocity of particles is updated. It should be noted that the acceleration coefficients, c_1 and c_2 , are different random values in the interval $[0, 1]$ and the inertia weight ω is defined as follows:

$$\omega = \omega_{\max} - \frac{\omega_{\max} - \omega_{\min}}{\text{Iter}_{\max}} \times \text{Iter} \quad (10)$$

where ω_{\max} is the final inertia weight factor, ω_{\min} is the initial inertia weight factor, Iter is the current iteration number, and Iter_{\max} is the maximum iteration number.

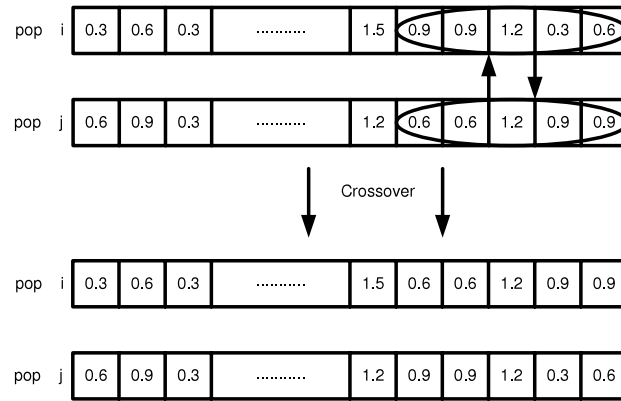


Fig. 3. A sample crossover operation.

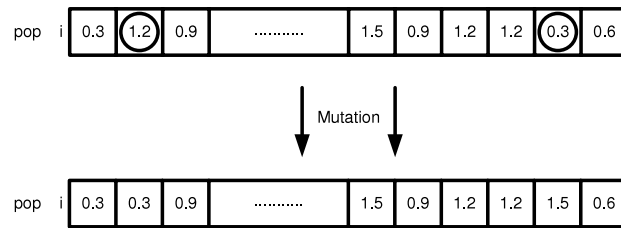


Fig. 4. A sample mutation operation.

As observed in (10), ω is to adjust the effect of the velocity in the previous iteration on the new velocity for each particle. Regarding the obtained velocity of each particle by (10), the position of particles can be updated for the next iteration using (11):

$$X_j^{k+1} = X_j^k + V_j^{k+1}. \quad (11)$$

The inertia weight factor is set as 0.9 and both the acceleration coefficients as 0.5 in this paper.

After this step, half of the population members continue the DPSO procedure and other half goes through the crossover and mutation operators. The first half continues their route at Step 7 and the second half goes through Step 6.

Step 6. (Apply crossover and mutation operators)

In this step, the crossover and mutation operators are applied to the half of the population members. This is done to increase the diversity of the optimizing variables to improve the local minimum problem. Figs. 3 and 4 show the operation of crossover and mutation operators.

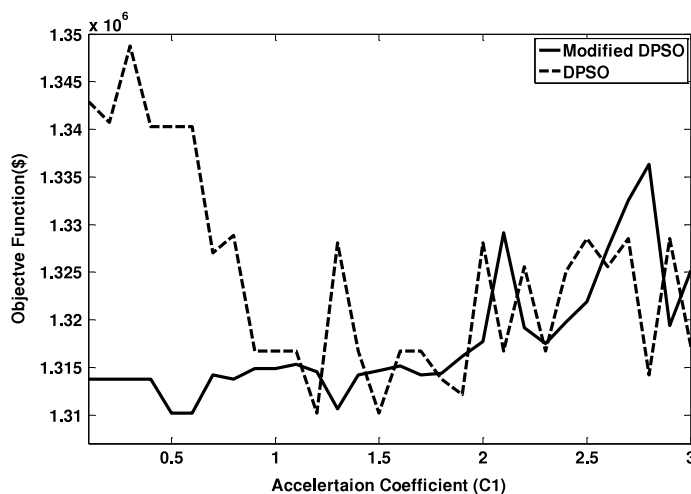
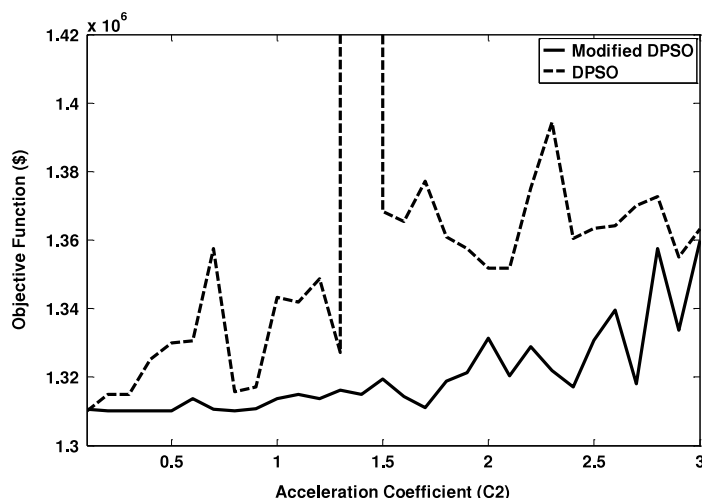
Step (Check convergence criterion)

If $\text{Iter} = \text{Iter}_{\max}$ or if the output does not change for a specific number of iterations, the program is terminated and the results are printed, else the program goes to Step 2.

4. Results

To validate the proposed method, two test systems are studied: the 12.5 kV 18-bus IEEE distribution system as case 1 and the 11 kV 37-bus distribution system connected to bus 2 of the Roy Billinton test system as case 2. It is assumed that the energy cost is 6 ¢/kWh. The installation cost of capacitors is assumed to be 4\$/kvar and the annual incremental cost is selected 8.75% of the installation cost. The available capacitors are considered as the 300 kvar banks. The number of years in the study timeframe is assumed to be 20 years.

To evaluate the proposed method, it is compared with four methods for capacitor planning, DPSO, GA [6–8], SA [12], and Discrete Nonlinear Programming (DNLP) [1]. DPSO is programmed as an m-file in Matlab. In order to simulate the rest of these optimization methods, the optimization tool in Matlab, called Optimtool, is used. This tool includes GA and SA, but for simulating DNLP, 'fminunc' [15] and 'fminsearch' [16], as nonlinear programming solvers in Optimtool, are modified by quantizing the optimizing variables (capacitor rating) in each step.

Fig. 5. OF versus acceleration coefficient c_1 .Fig. 6. OF versus acceleration coefficient c_2 .

4.1. Case 1

The 12.5 kV 18-bus IEEE distribution system is modified and used in this case. In this system, 16 buses are candidate for installing the capacitors. Therefore, the number of optimizing variables is 16. The population number is assumed to be about 15 times the number of optimizing variables. Hence, the number of population is selected as 250.

The robustness of MDPSO, with respect to changes of the PSO parameters, is studied and compared with DPSO. Fig. 5 shows the trend of the objective function versus an acceleration coefficient, c_1 in (9). During the computations, the rest of parameters are kept constant. Moreover, the initial values in both of the DPSO and MDPSO are assumed to be identical. As shown in Fig. 5, the changes of the objective function versus c_1 for MDPSO are lower than DPSO. The 'relative standard deviation' index (%RSD defined as the standard deviation divided by the average) is used to evaluate the robustness of methods. The lower this index is, the more robust a method will be. The %RSD of the objective function points (see Fig. 5) for MDPSO is %0.48 being lower than the %0.8 for DPSO.

In order to decrease these values more, a range of (0.1–2) for MDPSO and (0.7–3) for DPSO are assigned for this acceleration coefficient. These ranges reduce the %RSD to %0.3 and %0.6 for MDPSO and pure DPSO, respectively. The 'average' index is used to evaluate the accuracy of methods. The higher this index is, the more accurate a method will be. Given the average of the objective function points, \$1 317 621 for MDPSO and \$1 324 609 for DPSO, the higher accuracy of the proposed method over DPSO is illustrated. Similar to c_1 , the trend of the objective function versus c_2 is studied for MDPSO and pure DPSO (Fig. 6). It is observed that DPSO for $c_2 = 1.4$ does not satisfy the constraints. The higher accuracy of MDPSO over DPSO is seen in this figure. The %RSD of the MDPSO based on the objective function points is %0.97 and %0.6 for the c_1 ranges of (0.1–3) and (0.1–2), respectively.

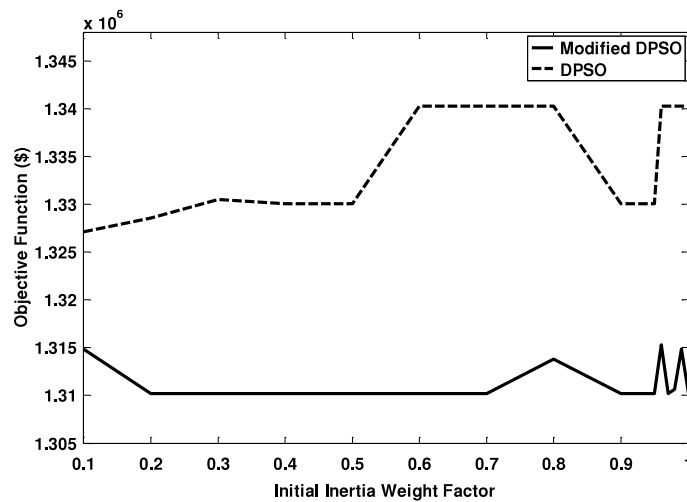


Fig. 7. OF versus initial weight factor ω_{\min} .

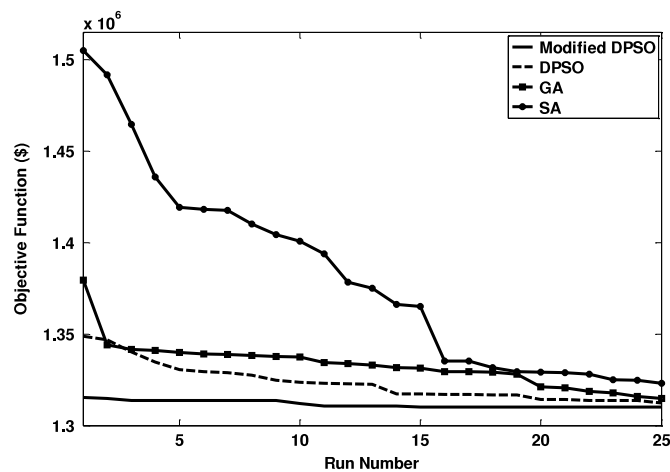


Fig. 8. A comparison of objective functions.

Fig. 7 depicts the trend of the objective function versus the initial weight factor, ω_{\min} . The %RSD of the objective function points is %0.13 and %0.40 for MDPSO and DPSO, respectively. This highlights the insensitivity of these optimization methods to this parameter. The average of the objective function points is \$1 311 178 for MDPSO and \$1 334 127 for DPSO which demonstrates the higher accuracy of the proposed technique. A range of (0.1–1) is appropriate for both of the methods, in which the objective function variation is negligible.

Similar to this procedure is performed for the final weight factor, ω_{\max} , and ranges of (0.4–1) and (0.6–1) are selected as the robustness range of this parameter for MDPSO and DPSO, respectively. In these ranges, %RSD for MDPSO is %0.65 and in pure DPSO is %0.85. Furthermore, the average objective function for MDPSO and pure DPSO is \$1 312 175 and \$1 331 140, respectively. This shows \$18 965 cost benefit by employing the proposed method.

As comprehended, both of the MDPSO and DPSO methods are insensitive to a wide range of their parameters. This robustness verifies the selection of these PSO-based algorithms as good options for solving the OASC problem. Particularly compared with DPSO, MDPSO is more robust and accurate.

After studying the robustness, with respect to changes of the PSO parameters, the robustness with respect to changes of the initial values is investigated. For this purpose, the MDPSO, DPSO, GA, and SA are run 25 times and the outputs are sorted by optimized Objective function value in Fig. 8. Table 1 shows a summary of the results.

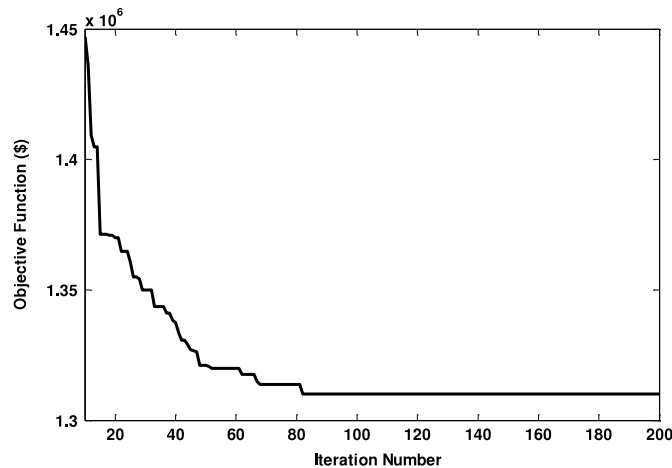
As observed in Table 1, MDPSO is the most accurate method compared with DPSO, GA and SA for capacitor planning in this case study (the error of average from the best point which is 1.3102×10^6 by MDPSO is \$1500 by DPSO is \$13 300, by GA is \$22 900, and by SA is \$57 600). This means \$11 800, \$21 400, and \$56 100 cost benefits are gained by employing MDPSO instead of DPSO, GA and SA, respectively. As observed in Fig. 8, the robustness of MDPSO is more than others (%RSD by MDPSO is %0.138 being lower than %0.766 by DPSO, %0.953 by GA, and %4.01 by SA).

Higher robustness and accuracy features highlight the priority of MDPSO over the other methods for capacitor planning.

Table 1

Comparison of optimization methods.

	Worst (\$)	Best (\$)	Average error (\$)	%RSD
SA	1.5049×10^6	1.33231×10^6	57 600	4.010
GA	1.3795×10^6	1.3147×10^6	22 900	0.953
DPSO	1.3488×10^6	1.3122×10^6	13 300	0.766
MDPSO	1.3153×10^6	1.3102×10^6	1 500	0.138

**Fig. 9.** Trend of OF versus iteration number.**Table 2**

Comparison of MDPSO, DPSO, GA, SA, and 'No Capacitor' state.

	Capacitors size (kvar)	Capacitor cost (\$)	Loss (kW)	Loss cost (\$)	Total cost (\$)
No Capacitor	0	0	332.1	1.849×10^6	1.849×10^6
SA	8100	6.243×10^4	234.4	1.3054×10^6	1.3678×10^6
GA	7800	6.012×10^4	229.4	1.2774×10^6	1.3375×10^6
DPSO	7800	6.012×10^4	227.3	1.2658×10^6	1.3259×10^6
MDPSO	7500	5.781×10^4	225.2	1.2540×10^6	1.3118×10^6

A comparison among the MDPSO, DPSO, GA, and SA along with the total cost with no installed capacitor is given in Table 2. In these heuristic methods, the median solution among the 25 runs is selected as the average value and is given in Table 2.

Table 2 illustrates that the total cost decreases from \$1 849 000 to \$1 311 800 by installing the capacitors (\$537 200 cost benefit). This underlines the importance of allocation and sizing of capacitors in a distribution system for minimizing the line loss. DNLP was also applied for capacitor planning, but it could not move from the initial values for many random initial values. This shows that the objective function has several local minima. Compared with other heuristic methods, MDPSO is demonstrated to be more accurate and robust for this case.

The trend of the objective function is depicted in Fig. 9 for the iteration number after 8. It should be noted that the objective function includes high penalty factors due to constraint violation in the first eight iterations.

The objective function value at the 9th iteration is \$1 620 558. This value decreases to \$1 310 202 at the 82nd iteration. Fig. 10 shows a comparison between the voltage profile before and after the installation of capacitors.

Before installation of capacitors, the bus voltage in four buses, 8, 24, 25, and 26 is lower than 0.95 pu which is unacceptable. As shown in this figure, the voltage profile has been increased in all buses to the standard range by installing the capacitors.

4.2. Case 2

The Roy Billinton test system is studied in this case as the second test system. This test system is shown in Fig. 11 and its characteristics are given in Table 3.

As shown, 22 loads are located in the test system. These are composed of 9 residential loads and 6 government loads at feeders F1, F3 and F4, 5 commercial loads at feeders F1 and F4, and 2 industrial loads at feeder F2.

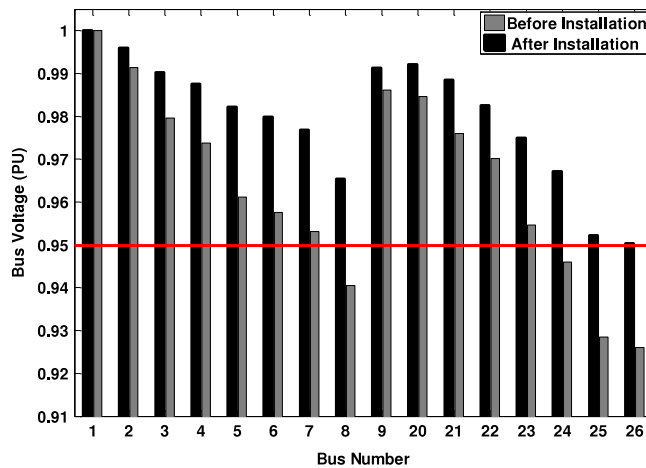


Fig. 10. Voltage profile before and after installation of capacitors.

Table 3

Characteristics of the test system.

Customer type	Load points	Load level MW
Residential	1–3, 10–12, 17–19	0.50
Commercial	6–7, 15–16, 22	0.45
Government	4–5, 13–14, 20–21	0.57
Industrial	8–9	1.10

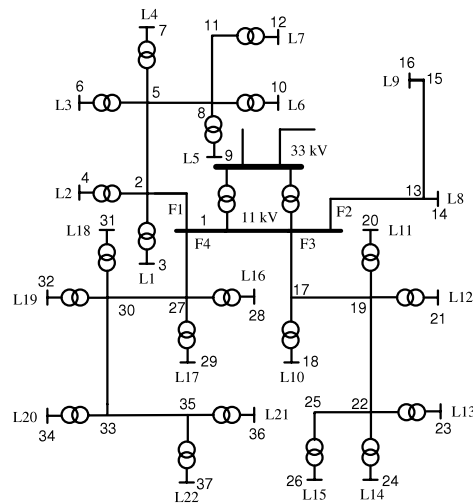


Fig. 11. Test distribution system in case 2.

The program is run 10 times using each of the optimization methods, MDPSO, DPSO, GA and SA. The results here are based on the median solution among these 10 runs.

Before installation of capacitors, the voltage at buses 36 and 37 is lower than 0.95 pu and the line loss is 363 kW. The voltage profile before and after installation of capacitors is shown in Fig. 12. The line loss decreases to 242.7 kW by installing capacitors (by MDPSO).

As shown in Fig. 12, the bus voltage at all buses has been increased to more than 0.95 pu by installing the capacitors. In order to evaluate the proposed method, the results are compared with DPSO, GA, SA, and 'No Capacitor' state (Table 4).

As shown in Table 4, the MDPSO demonstrates higher accuracy rather than DPSO, GA, and SA; the average total cost by MDPSO is \$1 413 800, by DPSO is \$1 602 900, by GA is \$1 443 200, and by SA is \$1 548 400. As mentioned, the analytical methods (e.g. DNLP) do not deal appropriately with the problem with several local minima. This is revealed in this case similar to case 1, the DNLP cannot move from its initial values for many random initial values.

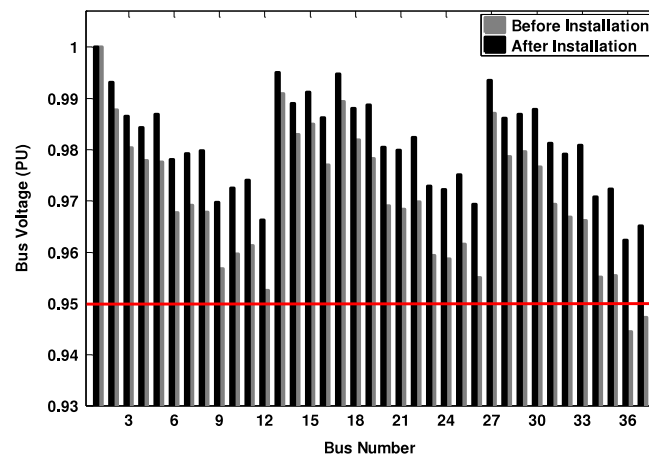


Fig. 12. Voltage profile before and after installation of capacitors.

Table 4

Comparison of MDPSO, DPSO, GA, SA, and 'No Capacitor' state.

	Capacitors size (kvar)	Capacitor cost (\$)	Loss kW	Loss cost (\$)	Total cost (\$)
No capacitor	0	0	363	2.0212×10^6	2.0212×10^6
SA	7500	5.781×10^4	267.7	1.4906×10^6	1.5484×10^6
GA	8400	6.475×10^4	247.6	1.3785×10^6	1.4432×10^6
DPSO	7500	5.781×10^4	277.5	1.5451×10^6	1.6029×10^6
MDPSO	8100	6.243×10^4	242.7	1.3514×10^6	1.4138×10^6

Similar to case 1, the importance of allocation and sizing of capacitors for minimizing the line loss is approved in this case so that the total cost decrease from \$2 021 200 to \$1 413 800 by installing the capacitors. The reasonable accuracy and robustness of the proposed MDPSO lead this method as a good choice for the capacitor planning problem.

If the required reactive power of all buses is provided by a capacitor located at the corresponding bus, the line loss is decreased to 239 kW. This reveals that the loss cannot be decreased to lower than 239 kW only by using capacitors; since, the rest of line loss is related to the active power.

5. Conclusions

In this paper, a Modified DPSO is presented to optimize the location and size of capacitors in a distribution system to minimize the line loss. The objective function is composed of the capacitors investment cost and the line loss which is converted into the genuine dollar. The bus voltage and the feeder current as constraints are maintained within the standard level.

Given the discrete nature of the capacitor planning problem, selection of a proper optimization method is important. The heuristic-based methods deal appropriately with the local minima. Among these methods, DPSO is employed in this paper. To increase the diversity of the optimizing variables, DPSO is developed by the crossover and mutation operators.

The proposed method is evaluated by two test systems: the 18-bus IEEE test system and the modified semi-urban distribution system connected to bus 2 of the Roy Billinton test system. The robustness and accuracy of the method are studied with respect to changes of the parameters and changes of the initial values. The results are compared with 'No Capacitor' state, DNLP as an analytical method, and three heuristic methods, DPSO, GA and SA. It is revealed that a high cost benefit is found by installing the capacitors. DNLP could never move from its initial values for many random choices of initial values. Compared with DPSO, GA and SA, MDPSO presents lower %RSD and average objective function which illustrates its higher robustness and accuracy for planning the capacitors.

References

- [1] A. Augugliaro, L. Dusanchoet, S. Mangione, Optimal capacitive compensation on radial distribution systems using nonlinear programming, *Electric Power Systems Research* 19 (2) (1990) 129–135.
- [2] Z.Q. Wu, K.L. Lo, Optimal choice of fixed and switched capacitors in radial distributors with distorted substation voltage, *IEE Proceedings-Generation, Transmission and Distribution* 142 (1) (1995) 24–28.
- [3] M.Y. Chao, Y.W. Chen, Fixed/switched type shunt capacitor planning of distribution systems by considering customer load patterns and simplified feeder model, *IEE Proceedings-Generation, Transmission and Distribution* 144 (6) (1997) 533–540.
- [4] R.A. Jabr, Optimal placement of capacitors in a radial network using conic and mixed integer linear programming, *Electric Power Systems Research* 78 (6) (2008) 941–948.

- [5] H.M. Khodr, F.G. Olsina, P.M. De Oliveira-De Jesus, J.M. Yusta, Maximum savings approach for location and sizing of capacitors in distribution systems, *Electric Power Systems Research* 78 (7) (2008) 1192–1203.
- [6] Gary Boone, Hsiao-Dong Chiang, Optimal capacitor placement in distribution systems by genetic algorithm, *International Journal of Electrical Power and Energy Systems* 15 (3) (1993) 155–161.
- [7] G. Levitin, A. Kalyuzhny, A. Shenkman, Optimal capacitor allocation in distribution systems using a genetic algorithm and a fast energy loss computation technique, *IEEE Transactions on Power Delivery* 15 (2) (2000) 623–628.
- [8] M.A.S. Masoum, M. Ladjavardi, A. Jafarian, Optimal placement, replacement and sizing of capacitor banks in distorted distribution networks by genetic algorithms, *IEEE Transactions on Power Delivery* 19 (4) (2008) 1794–1801.
- [9] M. Ladjavardi, M.A.S. Masoum, Genetically optimized fuzzy placement and sizing of capacitor banks in distorted distribution networks, *IEEE Transactions on Power Delivery* 23 (1) (2008) 449–456.
- [10] Ji-Pyng Chiou, Chung-Fu Chang, Ching-Tzong Su, Capacitor placement in large-scale distribution systems using variable scaling hybrid differential evolution, *International Journal of Electrical Power and Energy Systems* 28 (10) (2006) 739–745.
- [11] Chung-Fu Chang, Reconfiguration and capacitor placement for loss reduction of distribution systems by ant colony search algorithm, *IEEE Transactions on Power Systems* 23 (4) (2008) 1747–1755.
- [12] R.H. Liang, Y.S. Wang, Main transformer UTLC and capacitors scheduling by simulated annealing approach, *Electric Power Systems Research* 23 (7) (2001) 531–538.
- [13] J. Kennedy, R. Eberhart, Particle swarm optimization, in: *Proceedings of the 1995 IEEE International Conference on Neural Networks*, pp. 1942–1948.
- [14] Y. Del Valle, G.K. Venayagamoorthy, S. Mohagheghi, Particle swarm optimization: basic concepts, variants and applications in power systems, *IEEE Transactions on Evolutionary Computation* 12 (2) (2008) 171–195.
- [15] T.F. Coleman, Y. Li, On the convergence of reflective Newton methods for large-scale nonlinear minimization subject to bounds, *Mathematical Programming* 67 (2) (1994) 189–224.
- [16] J.C. Lagarias, J.A. Reeds, M.H. Wright, Convergence properties of the Nelder–Mead simplex method in low dimensions, *SIAM Journal on Optimization* 9 (1) (1998) 112–147.