Identification of the weakest bus in a distribution system with load uncertainties using reactive power margin

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***Abstract* — Recently, establishing reliable index to assess voltage stability condition of a stressed multi-node power transmission and distribution system is getting lot of attention. With the integration of renewable energy in power system the subject of voltage stability in distribution system is getting more interest. In this study, reactive power margin has been used as a stability index which appears as a reliable index to determine critical or weak node while considering the influence of load uncertainties. Effectiveness of reactive power margin index has been proved using a modified 16 bus primary distribution system. Some important results are presented in this paper.**

Keywords-voltage stability, distribution system, weak bus, reactive power margin.

I. INTRODUCTION

In recent years, increasing demand and lack of new generation and transmission networks have forced power transmission and distribution systems to operate closer to their security limits. In addition, due to the penetration of renewable energy based distributed generators; distribution systems are becoming more vulnerable to voltage instability problem. As a result some voltage stability indices have been proposed to identify weak nodes for protective measures, which helps proper placement of reactive power compensator and distributed generators for enhancement of system stability. Voltage instability or collapse is believed to be a local load bus problem, which can cause trouble to the entire system and depends mostly on load conditions in the system [1]. As most nodes are not voltage controlled, proper load modeling is more important in a distribution system than in a transmission system [2].

Load modeling for power system stability studies has always been a challenge for a number of reasons. The actual load below sub-transmission level consists of large varieties of components like thermostatic loads, resistive and inductive loads, induction motors, and lighting loads. Furthermore, number and type of loads varies continuously through time as different load components are switched on or off in response to residential, commercial and industrial activities. Other factors like change in weather may also cause highly unpredictable and irregular variations in the nature and amount of load. This statistical nature of load makes it very difficult to establish a load model that is generically applicable for power system studies [3]. To correctly analyze the voltage stability of a power system, suitable dynamic models are usually required based on nonlinear differential and algebraic equations. However, in many cases, static analysis tools can be used to identify influencing factors for long term voltage stability [4].

The influence of load modeling on different voltage stability indices has been worked out in few studies and it has been shown that with an increasing percentage of constant impedance load, indices related to P-V nose curves fails to measure the strength of nodes under stressed condition [5], [6]. In this paper, the impact of load uncertainties on reactive power margin as a voltage stability index has been comprehensively examined. Reactive power margins of load buses have been calculated for a primary distribution system to measure the strength of each bus with different compositions of load models. Results found in this study proves reactive power margin as a suitable index to identify the weakest bus irrespective of load pattern.

This paper is organized as follows. Section II gives a brief introduction to the weakest bus along with importance of its identification. Then section III describes the static voltage stability indices in use to identify bus strength. Section IV describes classification of static load models according to their voltage dependency. A brief introduction to test system and load compositions used in this study have been presented in section V. Section VI presents the simulation results along with some discussions and explanations of the findings. Finally section VII summarizes the major contributions and conclusions.

II. THE WEAKEST BUS

With the increased loading of transmission and distribution lines, voltage instability problem has become a concern and serious issue for power system planners and operators. The main challenge of this problem is to narrow down the locations where voltage instability could be initiated and to understand the origin of the problem. One effective way to narrow down the workspace is to identify weak buses in the systems, which are most likely to face voltage collapse. The weakest bus has been identified as the bus, which lacks reactive power support the most to defend against voltage

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collapse. Network configuration, R/X ratio of interconnections, load models, load directions, presence of generators and compensators are most influential factors of the strength of a bus in a distribution system. In turn, identifying weak buses can give correct information for the optimal reactive power planning involved that would decide which buses are the most severe and need to have new reactive power sources installed [7]. Ranking of bus based on strength has also been found useful in determining location for distributed generator to enhance loadability of the system [8]. Different voltage stability indices have been developed to determine the strength of load buses in a system, which has been explained in brief in the next section.

III. INDICES TO MEASURE BUS STRENGTH

A number of performance indices have been developed by the researchers to determine proximity of a load bus to voltage collapse [9]. Timely corrective actions can be taken only if an advance warning is issued to the system operator based on the measured stability indices.

A. Voltage Sensitivity Factor

Based on general concept, SF (sensitivity factor) index for a system represented by $F(z,\lambda)$ can be defined as $SF = \left\| \frac{dz}{d\lambda} \right\|$ [10]. When SF becomes large, the system turns insecure and ultimately collapses. Here the system voltages are checked with respect to the change in loading, which results in a Voltage sensitivity factor (VSF) calculated as $VSF = \left\| \frac{dV}{dP} \right\|$. High sensitivity means even small changes in loading causes large changes in the voltage magnitude, which indicates the weakness of a bus [11].

B. L-Index

L-index has been established as a fast voltage stability indicator for transmission system [12]. For an n-bus power system, buses can be separated into two groups: bringing all load buses to the head and denote them as α_L and putting the generator buses to the tail and term them as α_G i.e. $\alpha_L = \{1,2,.....,\ n_L - 1, n_L\}$, $\alpha_G = \{n_L+1, n_L+2,....,n-1,n\}$, where n_L is the number of load buses.

With a multi-node system,

$$I_{bus} = Y_{bus} \times V_{bus} \tag{1}$$

By arranging the load buses α_L and generator buses α_G as mentioned earlier, equation (1) can be written as

$$\begin{bmatrix} I_L \\ I_G \end{bmatrix} = \begin{bmatrix} Y_1 & Y_2 \\ Y_3 & Y_4 \end{bmatrix} \begin{bmatrix} V_L \\ V_G \end{bmatrix} \tag{2}$$

$$\begin{bmatrix} V_L \\ I_G \end{bmatrix} = H \begin{bmatrix} I_L \\ V_G \end{bmatrix} = \begin{bmatrix} Z_{LL} & F_{LG} \\ K_{GL} & Y_{GG} \end{bmatrix} \begin{bmatrix} I_L \\ V_G \end{bmatrix} \tag{3}$$

Where Z_{LL}, F_{LG}, K_{GL} and Y_{GG} are sub-block of matrix H ; V_G , I_G , V_L , I_L are voltage and current vector of generator and load buses respectively. For any load bus $j \in \alpha_L$ stability index L_j can be expressed as

$$L_j = \left| \frac{S_j^+}{Y_{jj}^{+*}.V_j^2} \right| \tag{4}$$

$$S_j^+ = \left(\sum_{k \in \alpha_L} \frac{Z_{jk}^*}{Z_{jj}^*} . \frac{S_k}{V_k} \right) V_j \tag{5}$$

Where S_k and V_k are complex power and complex voltage of node k respectively. The range of value is [0, 1]. Stability requires that $L_j < 1$ and must not be violated on a continuous basis. A global system indicator L describing the stability of the complete system is $L = \max(L_j)$. When L approaches 1.0; power system will approach voltage collapse. In practice L must be lower than a threshold value. The predetermined threshold value is specified at the planning stage depending on the system configuration and utility policy regarding quality of service.

C. Reactive Power Margin

Reactive power margin for a load bus is measured as a distance between the lowest MVAr point of Q-V curve and Voltage axis as shown in Fig. 1 [13],[14]. Thus reactive power margin indicates how further the loading on that particular bus can be increased before its loading limit is exhausted and voltage collapse takes place [11]. Reactive power margins are used in [15] to evaluate voltage instability problems for coherent bus groups. These margins are based on the reactive reserves on generators, SVCs and synchronous condensers that exhaust reserves in the process of computing a Q-V curve at any bus in a coherent group or voltage control area. In this study, the reactive power margin index is used to measure the strength of the buses of a primary distribution network for different sets of load models. The results have

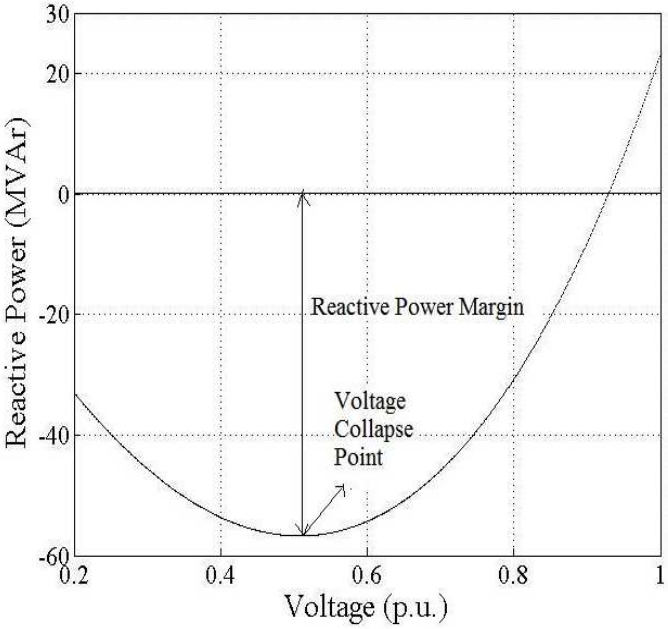


Figure 1. Example Q-V curve and reactive power margin.

been compared with two other established indices as mentioned earlier: VSF and L-index.

IV. Static Load Models

Most of the power system loads are connected to low-voltage or medium voltage distribution systems rather than to a high voltage transmission system. In transmission system, voltages are generally regulated by various control devices at the delivery nodes. Therefore in load flow calculation, loads can be represented by using constant power load models. In distribution systems, however, voltages differ widely along system feeders as there are less voltage control devices. So in load flow studies of distribution system, V-I characteristics of loads are more significant [2]. Load models are usually classified into two wide categories: static models and dynamic models. Static load models are relevant to load flow studies as these express active and reactive powers as functions of the bus voltages. They are typically classified as constant impedance, constant current and constant power load model.

In general, a static load model that represents the power relationship to voltage can be expressed by following exponential equations

$$P = P_0 \left(\frac{V}{V_0} \right)^{np} \tag{6}$$

$$Q = Q_0 \left(\frac{V}{V_0} \right)^{nq} \tag{7}$$

Here P_0 and Q_0 stand for real and reactive power consumed at a reference voltage V_0. The exponents np and nq depend on the load type, e.g., for constant power load models $np = nq = 0$, for constant current load models $np = nq = 1$ and for constant impedance load model $np = nq = 2$. Table 1 shows typical values of np and nq for several types of loads encountered in power systems [13], [14].

Along with exponential type as explained above, load can also be represented as polynomial load. Polynomial load model is a static load model that signifies the power-voltage relationship as a polynomial equation of voltage magnitude. It is usually referred as ZIP model as it is made up of different proportions of three types of load models: constant impedance (Z), constant current (I) and constant power (P). The real and reactive power characteristics of the ZIP load model are given by

$$P = P_0 \left[a_p \left(\frac{V}{V_0} \right)^2 + b_p \left(\frac{V}{V_0} \right) + c_p \right] \tag{8}$$

$$Q = Q_0 \left[a_q \left(\frac{V}{V_0} \right)^2 + b_q \left(\frac{V}{V_0} \right) + c_q \right] \tag{9}$$

TABLE 1: SAMPLE OF LOAD EXPONENTS

Load Component	**np**	**nq**
Fluorescent lighting	1	3
Resistance space heater	2	0
Room air conditioner	0.5	2.50
Incandescent lamp	1.54	0
Small industrial motors	0.1	0.6
Large industrial motors	0.05	0.5

Where $a_p + b_p + c_p = a_q + b_q + c_q = 1$ and P_0 and Q_0 stand for real and reactive power consumed at a reference voltage of V_0. In order to simulate various combinations of composite loads, the user can define the percentage co-efficient $a_p, b_p, c_p, a_q, b_q, c_q$ separately.

V. Test System and Load Compositions

In this study, 16-bus distribution system as shown in Fig. 2 is used. This system is a 23 kV primary distribution system with a total load of 28.7MW and 17.3MVAr, respectively. This system is a modified form of the one used in [16]. All the results presented in this paper were simulated with DIgSILENT PowerFactory 14.0 [17], a commercial tool and also have been verified using research analytical tool PSAT [18] and routines written in Matlab.

A wide variation of load composition is simulated through different sets of load model, which includes both extreme conditions and some intermediate conditions. In this paper, three types of extreme load models have been considered along with 3 intermediate combinations represented by ZIP load model. These six cases of load compositions have been listed in Table 2. Here extreme ZIP cases involve total existence of only one type of load i.e. constant power, constant current and constant impedance.

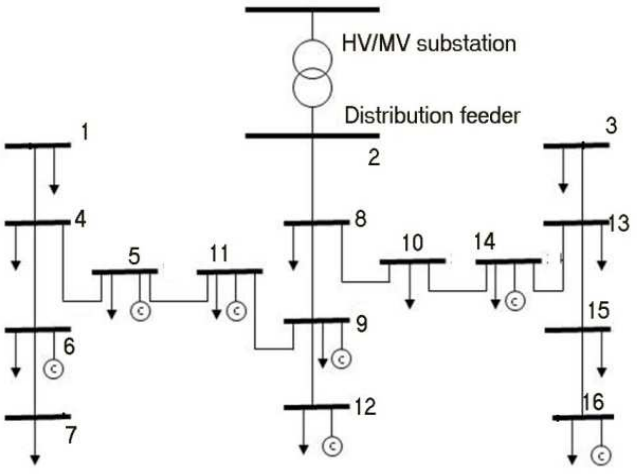


Figure 2. Single line diagram of test system.

TABLE 2: RANDOM LOAD COMPOSITION

Case	ZIP composition of loads		
	Z(%)	*I(%)*	*P(%)*
Extreme P	0	0	100
Extreme I	0	100	0
Extreme Z	100	0	0
Combination 1	0	50	50
Combination 2	30	20	50
Combination 3	80	20	0

TABLE 3: TYPICAL LOAD COMPOSITION [3]

Load Class	Load composition (%)		
	Residential	*Commercial*	*Industrial*
Resistive	25	14	5
Small Motor	75	51	20
Large Motor	0	0	56
Discharge Lighting	0	35	19

This study has then been extended to practical scenarios. It is unlikely that a utility can easily divide customer loads directly into load components as mentioned in Table 1. Rather doing that, utilities break their load into 3 compositions with the presence of different percentages of loads. The breakdown typically used by the utilities has been shown in Table 3 [3]. Here motors with power rating greater then 100hp has been treated as large motors, which are principal loads in industrial feeder. Residential and commercial loads are dominated by small motors representing air conditioner and water-pumps. Fluorescent lighting comprises 35% of commercial feeder, whereas incandescent lights are the major illumination loads for residential feeder.

VI. SIMULATION RESULTS AND DISCUSSION

Along with reactive power margin (Q_{margin} in MVAr), voltage sensitivity factor (V p.u. / MW) and L-index have also been measured to show the comparison in their performance with load type variation. Table 4 presents measured stability indices for three extreme types of loads i.e. constant power, constant current and constant impedance loads on all buses of the test distribution system. Table 5 shows the measured indices for 3 intermediate combinations with different proportions of loads as mentioned in Table 2. Both Tables include the measured stability indices for first three weak buses in the system. The lowest value of Q_{margin} and the highest value of VSF and L-index establish bus 7 as the weakest bus. For example, for constant power load models, lowest Q_{margin} 22.94MVAr has been found with bus 7, which establishes it as the weakest bus. The finding is in complete agreement with VSF and L-index which have highest values at, 0.53 and 0.22 respectively, to prove bus 7 as the weakest bus. VSF is found to be equally sensitive as reactive power margin while L-index is less sensitive in identifying a weak bus. However, for constant impedance load VSF and L-index fails to provide any information on the strength of a bus. But Q_{margin} still acts as good indicator of bus strength with those cases as seen from Table 4. The reason behind this observation can be explained with P-V curve obtained from continuation power flow.

In traditional study on steady state voltage stability, P-V curve is a commonly used tool, where nose point on the P-V curve is considered as the system collapse point. However, many researchers have pointed out that the real collapse point should be the Saddle Node Bifurcation (SNB) of the bifurcation curve but not the nose point of P-V curve. When all loads are constant power type, nose point coincides with the SNB point. But for other load types these points are usually not equal. Now, VSF and L-index both are based on traditional P-V curve studies where a higher value of these indices near collapse point indicates system instability. The bus with the highest VSF and L-index establishes itself as the weakest bus in the system. P-V curves for bus 5 with three extreme load types obtained from continuation power-flow. If the loads are modelled as constant power only, loadability margin is the lowest of all, which is found as 2.62 p.u.

TABLE 4: WEAK BUS DETECTION FOR EXTREME LOAD TYPES

Rank	Bus No.	Constant power			Constant current			Constant impedance		
		$Q_{margin}(MVAr)$	*VSF*	*L-index*	$Q_{margin}(MVAr)$	*VSF*	*L-index*	$Q_{margin}(MVAr)$	*VSF*	*L-index*
1	Bus 7	22.94	0.53	0.22	26.36	0.071	0.49	28.25	-	-
2	Bus 6	24.41	0.50	0.22	28.16	0.062	0.44	30.19	-	-
3	Bus 4	32.54	0.34	0.19	38.07	0.029	0.28	40.93	-	-

TABLE 5: WEAK BUS DETECTION FOR INTERMEDIATE LOAD TYPES

Rank	Bus No.	Combination 1			Combination 2			Combination 3		
		$Q_{margin}(MVAr)$	*VSF*	*L-index*	$Q_{margin}(MVAr)$	*VSF*	*L-index*	$Q_{margin}(MVAr)$	*VSF*	*L-index*
1	Bus 7	24.7	0.51	0.22	25.31	0.93	0.26	27.88	-	-
2	Bus 6	26.35	0.48	0.22	27	0.87	0.26	29.79	-	-
3	Bus 4	35.42	0.30	0.19	36.34	0.55	0.20	40.37	-	-

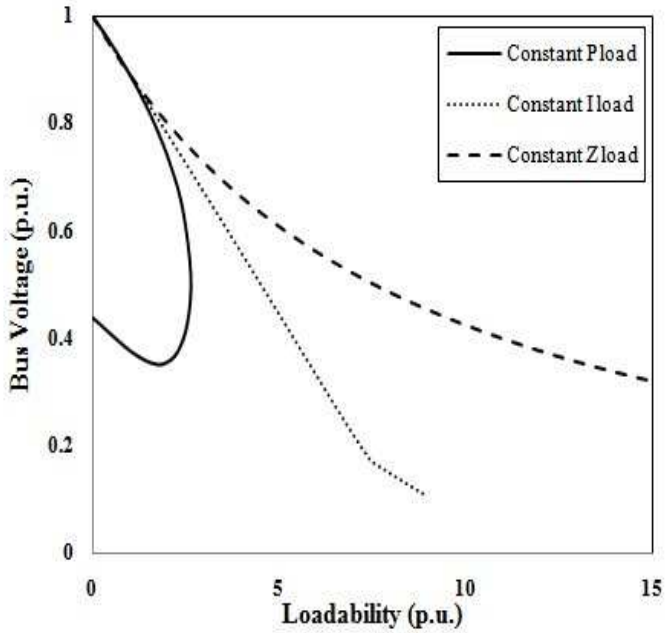


Figure 3. P-V curves for extreme types of loads.

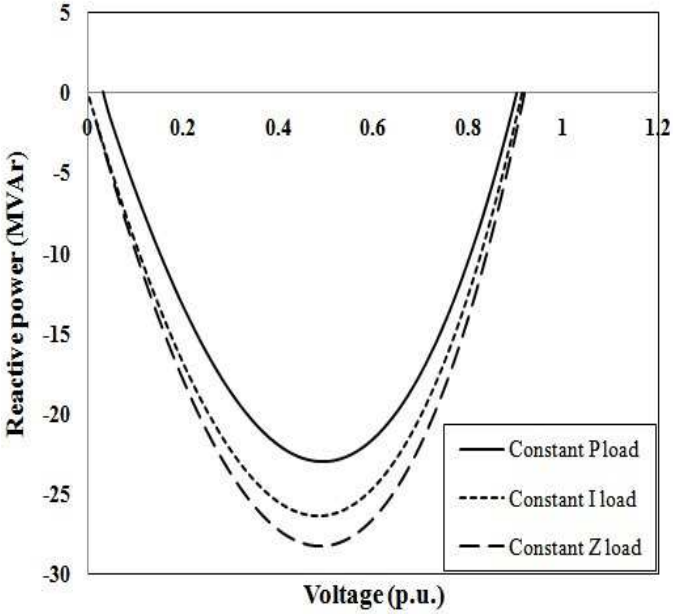


Figure 4. Q-V curves for extreme types of loads.

When load is considered as constant current only, loadability improves to 8.873p.u. With all the loads considered as constant impedance type, power flow could not find the collapse point while showing unlimited loading, which is an indication of absence of saddle node bifurcation point. But practically distribution system loading cannot be considered as unlimited. In static voltage stability, we often ignore thermal loading limit of feeders and conductors. With constant current and constant impedance type loads, it is obvious that the system will reach thermal limit much before bifurcation point. Thus, because of the absence of effective collapse point with constant impedance type load, VSF and L-index values are not obtainable. The same phenomenon happens with increasing percentage of Z part in intermediate load as shown for combination 3 where the combination has 80% constant Z type along with 20% constant I type load. It has also been reported in [5] where the results showed that there will be no SNB point if constant Z type load increases beyond 50% in a load combination. Further investigations are required to find out the reason behind this divergence in distribution system with constant Z type load. But in every case, Q_{margin} provides information to identify the weakest bus in the system. Q-V curve studies of bus 7, the weakest bus, with three extreme load types has been shown in Fig. 4. It shows that highest reactive power margin 28.25MVAr occurs with constant Z type load whereas constant P type load offers the lowest reactive power margin i.e. 22.94MVAr. With intermediate loads, it has been observed that with increasing penetration of Z type load, reactive power margin improves resulting in an improved stability of the system.

The work is then extended to practical scenario where the system is supplying demand to one of the three usual load compositions: residential, commercial and industrial (data taken from Table 3). Bus 7 has again been identified as the weakest bus with lowest reactive power margin among all the buses, followed by buses 6 and 4. A comparison of Q_{margin} for different load composition has been shown in Fig. 5. It has been observed that, the same distribution system offers improved stability when treated as a commercial feeder compared to a residential or an industrial feeder. For example, bus 7 (the weakest bus as per finding) has Q_{margin} of 26MVAr with commercial load, which is higher than with residential or industrial load. This observation is more explicable with loadability study. The test system offers highest loadability of 4.3p.u. with commercial load composition. Residential and industrial both load compositions offer loadability of 3.5p.u. From all the measured data, it can be observed that constant P type load offers the lowest loadability and reactive power margin among all the compositions taken into account. But whatever the load composition is, reactive power margin index works as a reliable index in identifying weakest bus in a system.

Identification of the weakest bus of a system leads to proper placement of compensating devices for loadability enhancement. Maximizing loadability has always been a good choice for a distribution system operator who wants to optimize their resources and maximize their profits. Here we verify the correctness of Q_{margin} for bus strength measurement by injecting a constant reactive power source, say 5MVAr, to each load bus and calculate resulting loadability affected by the injection of constant reactive power. The test results for constant power type load are shown in Fig. 6. Fig. 6 shows that with constant reactive power injection to weak buses like bus 7, 6, 4 and 5, loadability has been enhanced up-to an approximate value of 2.726 p.u. Reactive power injection at strong buses like buses 8 and 9 results in a loadability of around 2.708 p.u. which is lower than the previous case. This establishes weak bus as the proper place for injecting reactive power in the system.

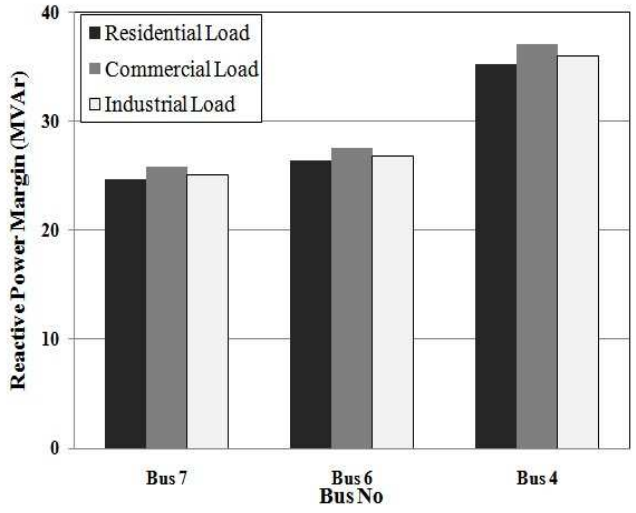


Figure 5. Reactive power margin with typical load composition.

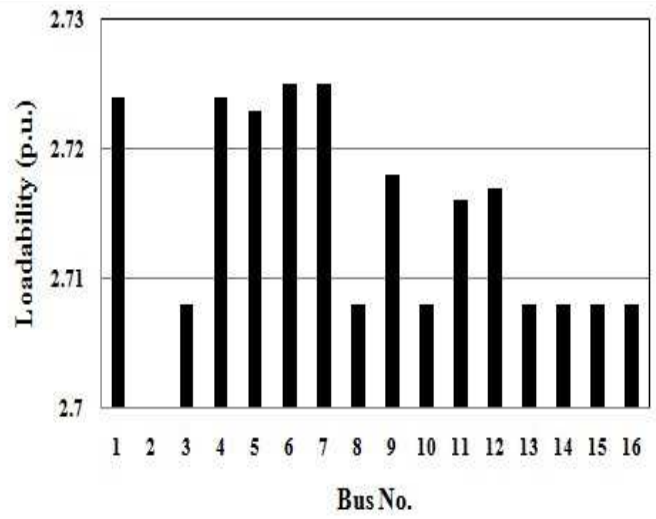


Figure 6. Loadability improvement with compensator placement on individual nodes.

VII. CONCLUSIONS

Though load model plays significant role in occurrence of voltage collapse and instability problems, development of load models is always a challenge to reflect the reality of a power system. So stability index, which can be used irrespective of load types, is required to identify the weakest bus and take protective measures. Results shown here prove that reactive power margin can be used as a reliable stability index to identify critical node with all kind of load uncertainties in a distribution system.

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